ELE339, Electronics I Laboratory

LAB 6 - Non-linear Elements and Distortion

Pre-Lab

Objective:
In this lab, we investigate why circuit elements with (weakly) non-linear characteristics cause (harmonic) distortion and how we can quantitatively express the severity of the distortion. To do so, we revisit an old acquaintance, the PN junction diode.

Introduction:
To simplify the analysis of circuits comprising non-linear elements, we typically replace the non-linear characteristics by piecewise linear approximations. This considerably simplifies the circuit analysis and leads to insightful conclusions, but the answers are only approximate. To express the severity of the non-linear characteristics, engineers frequently apply (Taylor) series approximations.
The PN junction diode represents a good example to illustrate this approach. We have learned that the relationship between junction current I_d and junction voltage V_d under forward bias conditions can be expressed as follows:

$$I_d = I_S \exp(\frac{V_d}{n_F V_T}) \qquad (1)$$

where n_F and V_T denote the emission coefficient and the thermal voltage, respectively. If the diode voltage varies as a sinusoid centered around a dc value, i.e.,

$$V_d(t) = V_{DC} + V_{AC}\sin(\omega_0 t) \qquad (2)$$

we can approximate the diode current I_d by a Taylor series expansion:

$$I_d(t) = I_{DC}[1 + k_1\sin(\omega_0 t) + k_2\sin(\omega_0 t)^2 + k_3\sin(\omega_0 t)^3 + ...] \qquad (3)$$

Consequently, the diode current I_d consists of not only a constant plus a linear term, i.e., $I_{DC}\, k_1\sin(\omega_0 t)$, but also components proportional to all higher order terms of the ac component of the diode voltage V_d. Since the n^{th} power of a sinusoid always contains a component, which is equal to the n^{th} multiple of the frequency of the input variable, i.e., $n\omega_0$, these higher order terms give rise to *harmonics*. The resulting signal distortion is typically expressed by the *Total Harmonic Distortion* (THD), which is defined as the *sum of the power of all harmonics divided by the total signal power*.
In a practical setting, where the distortion caused by a (weakly) non-linear element may be very small, the deviation from the linear response may hardly be visible. In such a case, engineers perform a mathematical transformation, which converts the *time domain* signal into the *frequency domain* (have you heard of this mapping function?), where the (weak) signal components at integer multiples of the *fundamental* frequency of the input signal are readily visible. As a matter of fact, your digital lab scopes can conveniently perform this signal mapping function.
To better illustrate the impact of a circuit non-linearity, we will apply a comparatively large sinusoidal signal of 10 kHz across a PN junction diode, or more specifically, across the base-emitter junction of an NPN transistor. This will visibly distort the output signal so that we can readily observe the distortion as a flattening and sharpening, respectively, of the peak and through of the output voltage.

The circuit we will use to investigate the distortion caused by the non-linear PN junction is shown in figure 1. At the operating frequency of 10kHz, we can write the output voltage of this circuit as $\mathbf{V_{out}=V_{CC}-I_cR_C}$. Since $I_c \approx I_e$, the ac component of the output varies as the (non-linear) base-emitter junction current.

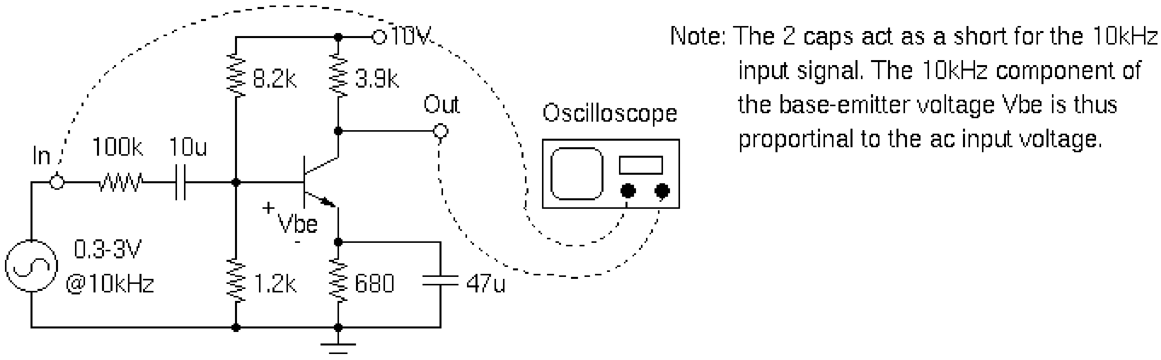

Figure 1: Test circuit for distortion analysis (Note: $V_{be} \approx V_{BE}+V_{ac}/120$).

Tasks:

1. Assume that equation (3) is terminated after the cubic term and express I_{DC}, k_1, k_2 and k_3 by the variables used in equations (1) and (2). (**pre-lab task**).
2. Approximate the function $\mathbf{F(\alpha)=e^{a \sin \alpha}}$ by using the first 4 terms of its Taylor series. Plot your approximation (e.g. in Matlab) for the 3 cases: a=0.1, a=0.25, a=0.75 ($0<\alpha<2\pi$). Apply the following trigonometric identities for the sine square and cube terms, respectively: $\sin^2 \alpha = {}^1/_2[1-\cos 2\alpha]$, $\sin^3 \alpha = {}^1/_4[3 \sin \alpha - \sin 3\alpha]$. (**pre-lab task**)
3. Determine the operating point parameters of the circuit shown in figure 1, i.e., V_C, V_B, V_E and I_E, via a PSpice simulation. The NPN transistor we are using in this lab is the 2N3904.
4. Use the transient analysis mode in PSpice and simulate the circuit for the 3 sinusoidal inputs: V_S=0.3V sin(ωt), V_S=1V sin(ωt) and V_S=3V sin(ωt), where ($\omega=2\pi$ x 10kHz).

Experimental

5. Build the circuit depicted in figure 1 on your Protoboard and verify its operating point parameters. Set V_s=0 for these measurements.
6. Use the waveform generator on your bench and mimic the 3 cases you simulated in task 4. Use the digital oscilloscope and observe input V_s and output V_{out} simultaneously (use the scope in AC coupling mode). Can you quantify the observed (time-domain) distortion on the oscilloscope?
7. Use the Math function on your oscilloscope to observe the frequency spectrum of your output voltage (use the FFT function with a *Hanning* window). Can you make sense of the displayed spectral plot? The TA will help you interpreting the display. Comment on how the numbers deduced from the oscilloscope compare to the theoretical results you derived in task 2, e.g. how closely does your circuit approach the expected ratio between the signal fundamental at 10kHz and its second and third harmonic component at 20 kHz and 30 kHz, respectively?